De Moivre's Theorem

Rule: $(r\ cis\ \theta)^n = r^n\ cis\ (n\theta)$

A proof of sorts: Consider $(r\ cis\ \theta)^4$.

$$\begin{aligned}(r\ cis\ \theta)^4 &= r\ cis\ \theta\ \times r\ cis\ \theta\ \times r\ cis\ \theta\ \times r\ cis\ \theta \\ &= r^2\ cis\ (\theta+\theta) \times\ r^2\ cis\ (\theta+\theta) \\ &= r^4\ cis\ (2\theta)\ \times cis\ (2\theta) \\ &= r^4\ cis\ (4\theta)\end{aligned}$$

Example 1: Use De Moivre's Theorem to work out $(4\ cis\ 60°)^3$.

$(4\ cis\ 60°)^3 = 4^3\ cis\ (3 \times 60°) = 64\ cis\ 180°$

Example 2: Instead of using repeated multiplication, use De Moivre's Theorem to work out $(2+5i)^4$.

- First convert $(2+5i)$ to polar form using Graphics Calculator:
- r = Abs $(2+5i)$ = 5.385
- θ = Arg $(2+5i)$ = 68.2°
- Therefore $(2+5i) = 5.385\ cis\ (68.2°)$
- $(2+5i)^4 = (5.385\ cis\ 68.2°)^4 = 5.385^4\ cis\ (68.2° \times 4)$
  $= 841\ cis\ (272.8°)$
- Now convert back to rectangular form:
- $841\ cis\ (272.8°) = 841\cos(272.8°) +\ i\ 841\sin(272.8°)$
  $= 41 - 840i$

* Note: The answer in rectangular form should be whole number integers (having expanded $(2+5i)^4$ which should not result in decimals), so round the values of $841\cos(272.8°)$ and $841\sin(272.8°)$ to whole numbers.

Delta Ex 32.3 pg 296 Q 1, 2. Extension Q 3 – 7.